J. Fluid Mech. (1965), *vol.* 22, *part* 1, *pp.* 39–47
Printed in Great Britain

The wave drag of a Hovercraft

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(Received 17 August 1964)

The wave drag of a Hovercraft is calculated as the horizontal force associated with a pressure distribution moving over the free water surface. A general formula is derived for the wave drag due to any pressure distribution moving over water of finite depth. This is then applied to rectangular and elliptical planform Hovercraft, and results are presented for a range of Froude numbers and water depths.

1. Introduction

A Hovercraft travelling over water sets up a train of induced waves moving at the speed of the craft. Energy is supplied to these waves by the pressure of the cushion on the surface, the horizontal component of the reaction appearing as 'wave drag' on the craft. Since a Hovercraft is largely separated from the water surface, it is possible to calculate the wave drag as the force associated with a pressure distribution which is assumed independent of the velocity of the craft.

Expressions for the wave resistance of a pressure distribution moving over a free liquid surface have been given by Havelock (1932). These are applicable to any distribution of pressure, for liquid of infinite depth. This work is here extended to cases when the water depth is finite, and is applied to specific pressure distributions, corresponding to the cushion pressure distributions of Hovercraft.

2. Analysis

Consider an infinite sea of uniform depth h. Let rectangular Cartesian axes Ox, Oy and Oz be fixed relative to a pressure distribution moving over the free surface, with the origin in the undisturbed surface, the x-axis in the direction of travel, and the z-axis vertically upwards. Let the pressure distribution $p(x,y)$ move with constant velocity c, and let $\eta(x,y)$ be the vertical surface disturbance.

Irrotational flow is assumed. The problem is made determinate by assuming a frictional force on each particle of liquid which is proportional to the velocity of that particle relative to the undisturbed liquid (Rayleigh 1883). The coefficient of this force (μ), which is defined as the frictional force per unit mass divided by the relative velocity of the particle, is made to approach zero. It can be shown that this device is not inconsistent with irrotational flow in the liquid.

Following Havelock, take a possible form for the velocity potential, and find the surface pressure to which it corresponds. Consider the function

$$\phi = \int_{-\pi}^{\pi}\int_{0}^{\infty} \frac{F(k)\cosh[k(z+h)]\,e^{ikw}k\,dk\sec\theta\,d\theta}{k-k_0\sec^2\theta\tanh kh+i\mu\sec\theta}, \tag{2.1}$$

where $$w = x\cos\theta + y\sin\theta \quad \text{and} \quad k_0 = g/c^2.$$

It can be verified that this satisfies Laplace's equation and the fixed boundary condition

$$(\partial\phi/\partial z)_{z=-h} = 0. \tag{2.2}$$

Also the corresponding surface pressure distribution will be shown to have a simple form (2.6), which allows the deduction of the velocity potential corresponding to an arbitrary pressure distribution.

The assumption is made that the slope of the surface and η/h are small. The surface velocity condition is then

$$c(\partial\eta/\partial x) = (\partial\phi/\partial z)_{z=0}. \tag{2.3}$$

Hence $$\eta = \frac{-i}{c}\int_{-\pi}^{\pi}\int_0^{\infty}\frac{F(k)\sinh kh\, e^{ikw} k\,dk\sec^2\theta\,d\theta}{k - k_0\sec^2\theta\tanh kh + i\mu\sec\theta}. \tag{2.4}$$

The surface pressure is given by

$$p/\rho = -c\partial\phi/\partial x - g\eta + uc\phi. \tag{2.5}$$

Therefore $$p = -ic\rho\int_{-\pi}^{\pi}\int_0^{\infty} F(k)\cosh kh\, e^{ikw} k\,dk\,d\theta,$$

$$= -2\pi ic\rho\int_0^{\infty} kF(k)\cosh kh\,\mathrm{J}_0(kr)\,dk, \tag{2.6}$$

where $r^2 = x^2 + y^2$ and J_0 is a Bessel function of the first kind. But

$$p(r) = \int_0^{\infty}\int_0^{\infty} p(\alpha)\,\mathrm{J}_0(k\alpha)\,\alpha\,d\alpha\,\mathrm{J}_0(kr)\,k\,dk, \tag{2.7}$$

(Havelock 1932, equation (24); Watson 1923).

Therefore from (2.1), (2.6) and (2.7), the velocity potential for a radially symmetric pressure distribution $p(r)$ is given by

$$\phi = \frac{i}{2\pi c\rho}\int_{-\pi}^{\pi}\int_0^{\infty}\frac{f(k)\{\cosh k(z+h)/\cosh kh\}\, e^{ikw} k\,dk\sec\theta\,d\theta}{k - k_0\sec^2\theta\tanh kh + i\mu\sec\theta}, \tag{2.8}$$

where $$f(k) = \int_0^{\infty} p(\alpha)\,\mathrm{J}_0(k\alpha)\,\alpha\,d\alpha.$$

If the pressure distribution is concentrated at the origin to produce a finite force P,

$$f(k) = P/2\pi. \tag{2.9}$$

So, generalizing for any continuous pressure distribution,

$$\phi = \frac{i}{4\pi^2 c\rho}\int_S\int_{-\pi}^{\pi}\int_0^{\infty}\frac{\{\cosh k(z+h)/\cosh kh\}\, e^{ik\overline{w}} k\,dk\sec\theta\,d\theta\, p(x', y')\,dS}{k - k_0\sec^2\theta\tanh kh + i\mu\sec\theta}, \tag{2.10}$$

where $\overline{w} = (x - x')\cos\theta + (y - y')\sin\theta$ and $\int_S dS$ indicates the integral taken over the surface S.

The wave resistance is given by

$$R = \lim_{\mu\to 0}\mu\rho\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi\frac{\partial\phi}{\partial z}\,dx\,dy \tag{2.11}$$

taken over the surface $z = 0$ (Lamb 1926). After some manipulation, taking the real part, the velocity potential becomes

$$\phi_{z=0} = \frac{1}{4\pi^2 c\rho}\int_{-\pi}^{\pi}\int_0^{\infty} (\mathrm{CC}\,F_1 + \mathrm{SC}\,F_2 + \mathrm{CS}\,F_3 + \mathrm{SS}\,F_4)\,k\,dk\,d\theta, \tag{2.12}$$

where

$$\left.\begin{aligned} \mathrm{CC} &= \cos(kx\cos\theta)\cos(ky\sin\theta), \quad \mathrm{SC} = \sin(kx\cos\theta)\cos(ky\sin\theta),\\ \mathrm{CS} &= \cos(kx\cos\theta)\sin(ky\sin\theta), \quad \mathrm{SS} = \sin(kx\cos\theta)\sin(ky\sin\theta),\end{aligned}\right\} \tag{2.13}$$

$$\left.\begin{aligned} F_1 &= \{(k - k_0\sec^2\theta\tanh kh)\,Q_e + \mu\sec\theta\,P_e\}\,D_F,\\ F_2 &= \{-(k - k_0\sec^2\theta\tanh kh)\,P_e + \mu\sec\theta\,Q_e\}\,D_F,\\ F_3 &= \{(k - k_0\sec^2\theta\tanh kh)\,Q_0 + \mu\sec\theta\,P_0\}\,D_F,\\ F_4 &= \{(k - k_0\sec^2\theta\tanh kh)\,P_0 - \mu\sec\theta\,Q_0\}\,D_F,\\ D_F &= \sec\theta/\{(k - k_0\sec^2\theta\tanh kh)^2 + \mu^2\sec^2\theta\},\end{aligned}\right\} \tag{2.14}$$

and

$$\left.\begin{aligned} P_e &= \int_S p(x',y')\cos(kx'\cos\theta)\cos(ky'\sin\theta)\,dS,\\ P_0 &= \int_S p(x',y')\sin(kx'\cos\theta)\sin(ky'\sin\theta)\,dS,\\ Q_e &= \int_S p(x',y')\sin(kx'\cos\theta)\cos(ky'\sin\theta)\,dS,\\ Q_0 &= \int_S p(x',y')\cos(kx'\cos\theta)\sin(ky'\sin\theta)\,dS,\end{aligned}\right\} \text{the integrals being taken over the surface } S. \tag{2.15}$$

Similarly,

$$\left(\frac{\partial\phi}{\partial z}\right)_{z=0} = \frac{1}{4\pi^2 c\rho}\int_{-\pi}^{\pi}\int_0^{\infty} (\mathrm{CC}\,G_1 + \mathrm{SC}\,G_2 + \mathrm{CS}\,G_3 + \mathrm{SS}\,G_4)\,k\,dk\,d\theta, \tag{2.16}$$

where the G's are the same as the F's except that

$$D_G = k\tanh kh\sec\theta/\{(k - k_0\sec^2\theta\tanh kh)^2 + \mu^2\sec^2\theta\}.$$

From the Fourier integral theorem it can be shown that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(x,y)\,G(x,y)\,dx\,dy = 4\pi^2\int_{-\pi}^{\pi}\int_0^{\infty}(F_1G_1 + F_2G_2 + F_3G_3 + F_4G_4)\,k\,dk\,d\theta, \tag{2.17}$$

where $$F(x,y) = \int_{-\pi}^{\pi}\int_0^{\infty}(\mathrm{CC}\,F_1 + \mathrm{SC}\,F_2 + \mathrm{CS}\,F_3 + \mathrm{SS}\,F_4)\,k\,dk\,d\theta$$

and

$$G(x,y) = \int_{-\pi}^{\pi}\int_0^{\infty}(\mathrm{CC}\,G_1 + \mathrm{SC}\,G_2 + \mathrm{CS}\,G_3 + \mathrm{SS}\,G_4)\,k\,dk\,d\theta \quad \text{(Havelock 1932).}$$

Thus $$\begin{aligned} R &= \lim_{\mu\to 0}\frac{\mu}{4\pi^2c^2\rho}\int_{-\pi}^{\pi}\int_0^{\infty}(F_1G_1 + F_2G_2 + F_3G_3 + F_4G_4)\,k\,dk\,d\theta,\\ &= \lim_{\mu\to 0}\frac{\mu}{4\pi^2c^2\rho}\int_{-\pi}^{\pi}\int_0^{\infty}\frac{(P_e^2 + P_0^2 + Q_e^2 + Q_0^2)(\tanh kh)\,k^2\,dk\sec^2\theta\,d\theta}{(k - k_0\sec^2\theta\tanh kh)^2 + \mu^2\sec^2\theta}.\end{aligned} \tag{2.18}$$

The above limit is non-zero only when

$$k-k_0\sec^2\theta\tanh kh = 0$$

has a real positive root (denoted k_1). For this to exist,

$$k_0\,h\sec^2\theta > 1.$$

Thus

$$R = \lim_{\mu\to 0}\frac{i}{2\pi^2c^2\rho}$$

$$\times\int_{\theta_0}^{\frac{1}{2}\pi}\left(\int_0^\infty \frac{(P_e^2+P_0^2+Q_e^2+Q_0^2)(\tanh kh)\,k^2\,dk}{(k_1-k_0\sec^2\theta\tanh k_1h)+(k-k_1)(1-k_0h\sec^2\theta\,\mathrm{sech}^2\,k_1h)+\ldots+i\mu\sec\theta}\right.$$

$$\left.-\int_0^\infty \frac{(P_e^2+P_0^2+Q_e^2+Q_0^2)(\tanh kh)\,k^2\,dk}{(k_1-k_0\sec^2\theta\tanh k_1h)+(k-k_1)(1-k_0h\sec^2\theta\,\mathrm{sech}^2\,k_1h)+\ldots-i\mu\sec\theta}\right)$$

$$\times\sec\theta\,d\theta, \quad (2.19)$$

where $\theta_0 = 0$ when $k_0h > 1$ or $c^2 < gh$.

$= \cos^{-1}(k_0h)^{\frac{1}{2}}$ when $k_0h < 1$ or $c^2 > gh$.

To evaluate the integrals with respect to k, consider the contour integrals enclosing the 1st and 4th quadrants and the positive half of the complex k-plane. Then

$$R = \frac{1}{\pi c^2\rho}\int_{\theta_0}^{\frac{1}{2}\pi}\frac{k_1^2\tanh k_1h\sec\theta\,(P_e^2+P_0^2+Q_e^2+Q_0^2)\,d\theta}{1-k_0h\sec^2\theta\,\mathrm{sech}^2\,k_1h}. \quad (2.20)$$

For $h\to\infty$, $k_1\to k_0\sec^2\theta$, and this reduces to the result obtained by Havelock, namely

$$R = \frac{k_0^2}{\pi c^2\rho}\int_0^{\frac{1}{2}\pi}(P_e^2+P_0^2+Q_e^2+Q_0^2)\sec^5\theta\,d\theta. \quad (2.21)$$

3. Applications

The pressure distribution due to a Hovercraft can normally be represented by a uniform pressure over the area of the cushion. Two cases are considered in detail; craft with rectangular and with elliptical cushion planforms.

(1) *Rectangular craft*

In this case from (2.15)

$$P_e = p_c\int_{-a}^{a}\cos(k_1x\cos\theta)\,dx\int_{-b}^{b}\cos(k_1y\sin\theta)\,dy,\quad P_0 = Q_e = Q_0 = 0. \quad (3.1)$$

Using the result $k_1 = k_0\sec^2\theta\tanh k_1h$,

$$R = \frac{16p_c^2}{\pi c^2\rho k_0}\int_{\theta_0}^{\frac{1}{2}\pi}\frac{\sin^2(k_1a\cos\theta)\sin^2(k_1b\sin\theta)\,d\theta}{k_1\sin^2\theta\cos\theta\,(1-k_0h\sec^2\theta\,\mathrm{sech}^2\,k_1h)}. \quad (3.2)$$

Putting $k_1/k_0 = K, b/a = A, h/2a = H, c(2ga)^{-\frac{1}{2}} = F_R$,

$$\frac{R}{W}\frac{\rho g\sqrt{S}}{p_c} = \frac{16F_R^2}{\pi\sqrt{A}}\int_{\theta_0}^{\frac{1}{2}\pi}\frac{\sin^2(K/2F_R^2\cos\theta)\sin^2(KA/2F_R^2\sin\theta)\,d\theta}{K\sin^2\theta\cos\theta\,(1-(H/F_R^2)\sec^2\theta\,\mathrm{sech}^2\,KH/F_R^2)}, \quad (3.3)$$

where $$K-\sec^2\theta\tanh KH/F_R^2 = 0, \quad S = 4ab, \quad W = p_c S$$

$$\theta_0 = 0 \quad \text{when} \quad H/F_R^2 > 1,$$

and $$\theta_0 = \cos^{-1}(H^{\frac{1}{2}}/F_R) \quad \text{when} \quad H/F_R^2 < 1.$$

For deep water (H large) this becomes

$$\frac{R}{W}\frac{\rho g\sqrt{S}}{p_c} = \frac{16F_R^2}{\pi\sqrt{A}}\int_0^{\frac{1}{2}\pi}\frac{\sin^2([1/2F_R^2]\sec^2\theta\cos\theta)\sin^2([A/2F_R^2]\sec^2\theta\sin\theta)\,d\theta}{\sin^2\theta\sec\theta}. \tag{3.4}$$

(2) *Elliptical craft*

$$P_e = \int_S p_c\cos(kx'\cos\theta)\cos(ky'\sin\theta)\,dS$$

Putting $x' = r\cos\phi$, $y' = (b/a)r\sin\phi$,

$$P_e = \frac{b}{a}p_c\int_0^a\int_0^{2\pi}\cos(kr\cos\phi\cos\theta)\cos\left(\frac{kb}{a}r\sin\phi\sin\theta\right)d\phi r\,dr$$

$$= 2\pi p_c\frac{b}{a}\int_0^a J_0(kr\{\cos^2\theta+(b/a)^2\sin^2\theta\}^{\frac{1}{2}})\,r\,dr$$

$$= \frac{2\pi p_c b}{k\{\cos^2\theta+(b/a)^2\sin^2\theta\}^{\frac{1}{2}}}J_1(ka\{\cos^2\theta+(b/a)^2\sin^2\theta\}^{\frac{1}{2}}), \tag{3.5}$$

$$P_0 = Q_e = Q_0 = 0.$$

Thus

$$R = \frac{4\pi p_c^2 b^2}{c^2\rho}\int_{\theta_0}^{\frac{1}{2}\pi}\frac{\tanh k_1 h\sec\theta\, J_1^2(ka\{\cos^2\theta+(b/a)^2\sin^2\theta\}^{\frac{1}{2}})}{\{\cos^2\theta+(b/a)^2\sin^2\theta\}(1-k_0 h\sec^2\theta\,\mathrm{sech}^2 k_1 h)}d\theta, \tag{3.6}$$

and

$$\frac{R}{W}\frac{\rho g\sqrt{S}}{p_c} = \frac{2\sqrt{(\pi)}A^{\frac{1}{2}}}{F_R^2}\int_{\theta_0}^{\frac{1}{2}\pi}\frac{KJ_1^2\{(K/2F_R^2)\cos\theta(1+A^2\tan^2\theta)^{\frac{1}{2}}\}}{\cos\theta(1+A^2\tan^2\theta)\{1-(H/F_R^2)\sec^2\theta\,\mathrm{sech}^2 KH/F_R^2)\}}d\theta, \tag{3.7}$$

where $S = \pi ab$.

For deep water, this becomes

$$\frac{R}{W}\frac{\rho g\sqrt{S}}{p_c} = \frac{2\sqrt{(\pi)}A^{\frac{1}{2}}}{F_R^2}\int_0^{\frac{1}{2}\pi}\frac{J_1^2\{(1/2F_R^2)\sec\theta(1+A^2\tan^2\theta)^{\frac{1}{2}}\}}{\cos^3\theta(1+A^2\tan^2\theta)}d\theta. \tag{3.8}$$

Similarly, it is possible to derive formulae corresponding to non-uniform pressure distributions. Results for these, and for the above two cases, are presented in the following section.

4 Results

Figures 1–9 show the non-dimensional wave drag coefficient plotted against Froude number, for a variety of craft configurations, in deep and in shallow water. The following general features can be seen:

(1) The drag coefficient rises to a maximum at 'hump speed', corresponding to a Froude number of 0·5–1·0 in deep water. Other peaks, sometimes of greater height, occur at lower speeds.

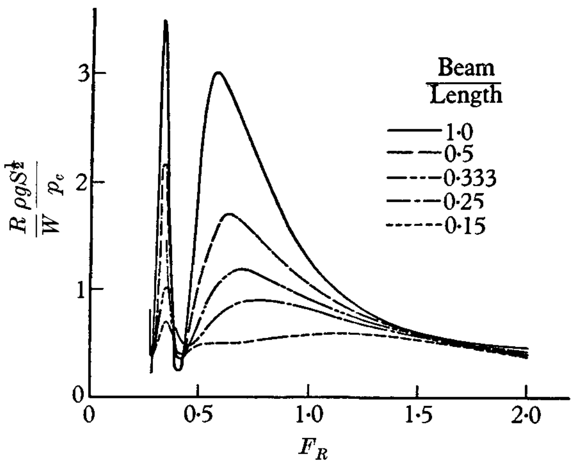

FIGURE 1. Wave drag of a rectangular planform Hovercraft in deep water.

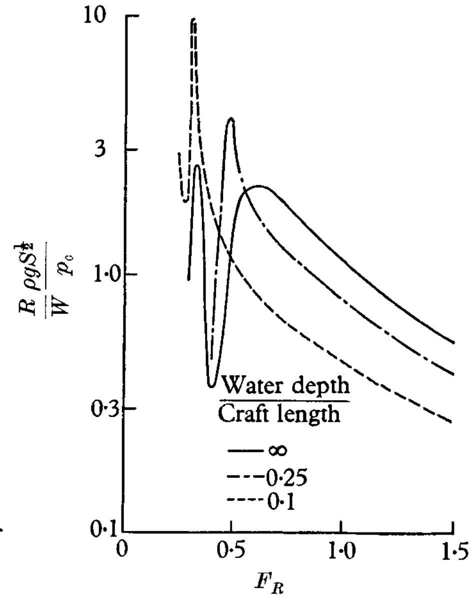

FIGURE 2. Wave drag of a rectangular Hovercraft in shallow water. Beam/length = 0·667.

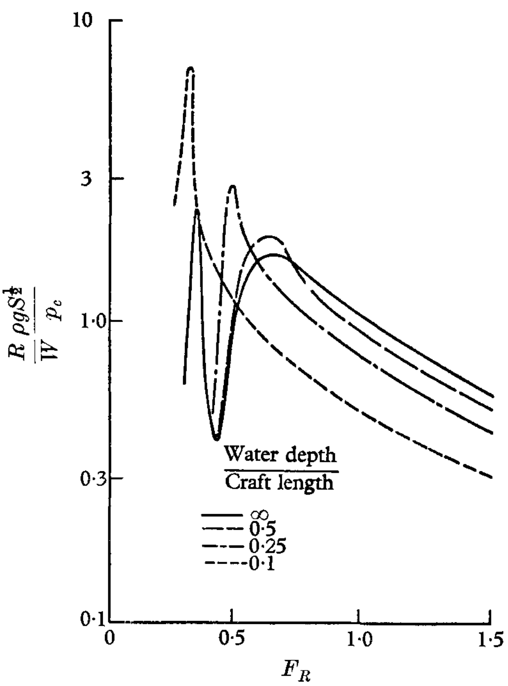

FIGURE 3. Wave drag of a rectangular planform Hovercraft in shallow water. Beam/length = 0·5.

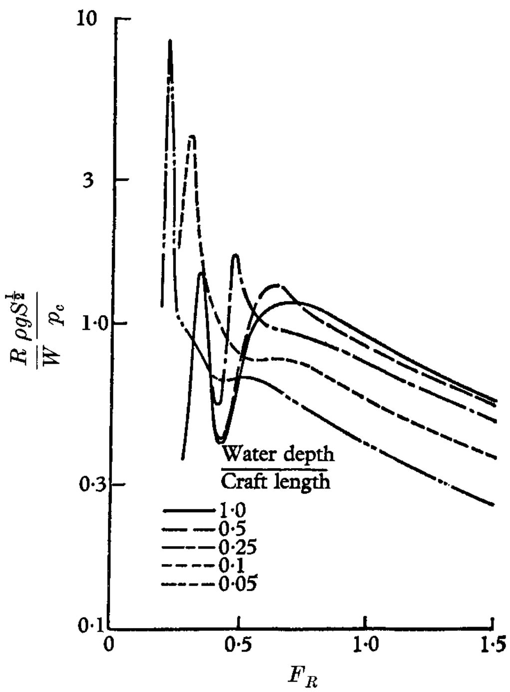

FIGURE 4. Wave drag of a rectangular planform Hovercraft in shallow water. Beam/length = 0·333.

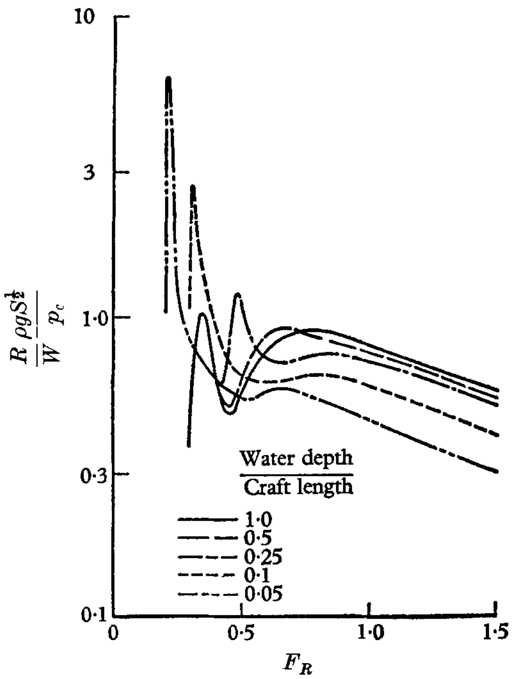

FIGURE 5. Wave drag of a rectangular planform Hovercraft in shallow water. Beam/length = 0·25.

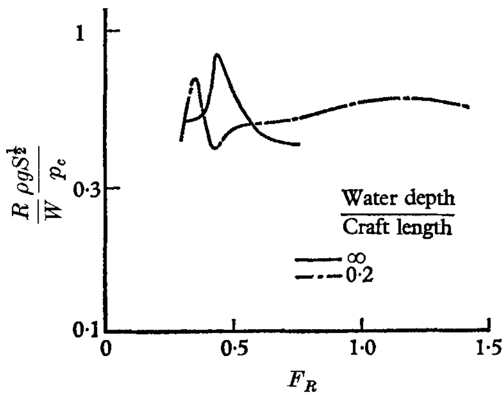

FIGURE 6. Wave drag of a rectangular planform Hovercraft in shallow water. Beam/length = 0·15.

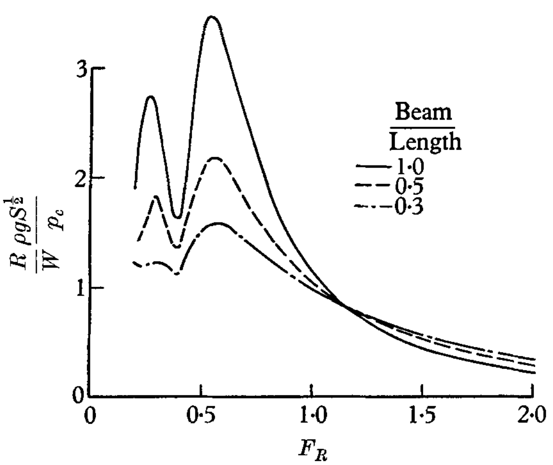

FIGURE 7. Wave drag of an elliptical planform Hovercraft in deep water.

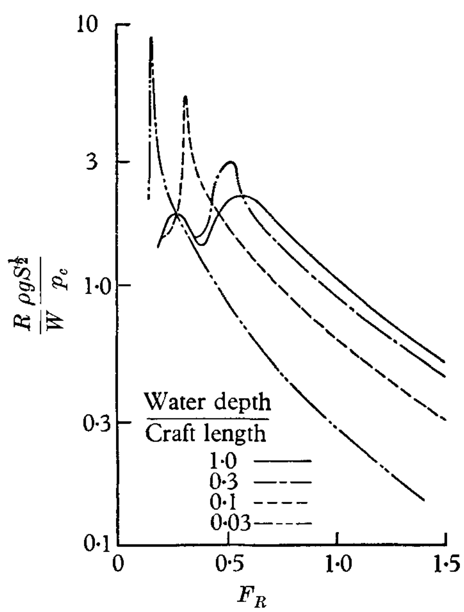

FIGURE 8. Wave drag of an elliptical planform Hovercraft in shallow water. Beam/length = 0·5.

(2) In shallow water, the hump speed moves to lower Froude numbers, given approximately by $(h/l)^{\frac{1}{2}}$, where l is the craft length. The peak value of the drag rises indefinitely as the depth decreases.

Figures 1–6 give the wave drag in deep and shallow water for a rectangular craft, while figures 7 and 8 give similar information for an elliptical craft. Comparing the deep-water results, it can be seen that for a given value of beam/length ratio the elliptical craft has higher values of hump drag, but the rectangular craft has higher values at the lower speed peaks. The curves have not been plotted for lower Froude numbers, because of the large number of points needed to define them, and the doubtful validity of the theory in these regions. In shallow water the rectangular planform craft shows greater fluctuations in wave drag than does the elliptical craft (figures 3 and 8).

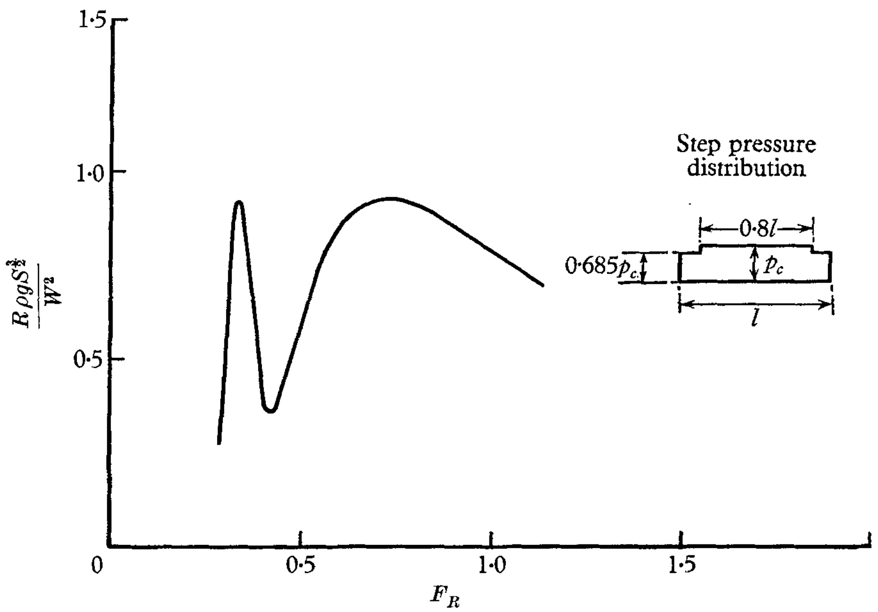

FIGURE 9. Wave drag of a rectangular planform Hovercraft in deep water. Step pressure distribution. Beam/length = 0·267.

A check on these results is provided by the work of Newman & Poole (1962). This provides similar results for canals of finite width, and appears to give identical results as the width becomes very large.

Figure 9 shows the drag of a Hovercraft with a step fore and aft pressure distribution. This might correspond to a craft with 'double curtains', in which two jets are used, one inside the other. In spite of the large number of points calculated, no double peaks or other unusual features are visible. The result is closely similar to that for a rectangular craft with uniform pressure distribution.

In the previous analysis it has been assumed that the wave slope is everywhere small and that the wave amplitude is small compared with the water depth. For small water depths or Froude numbers, these conditions are not met, leading to the unrealistically high drag peaks shown in the figures. By considering the maximum steepness attained by waves before breaking, Hogben (1964) has deduced limiting values for wave drag. These are only strictly applicable to two-dimensional waves, but do appear to give plausible results as far as it has been possible to check them.

Direct experimental confirmation of the above results is not available. However, estimates of the total drag using them have proved remarkably accurate.

The computation of the results was performed under the direction of Mr G. L. Webb. The author also wishes to acknowledge the helpful criticisms of Prof. W. A. Mair, and of his colleagues.

REFERENCES

HAVELOCK, T. H. 1932 The theory of wave resistance. *Proc. Roy. Soc.* A, **138**, 339–48.

HOGBEN, N. 1964 An investigation of hovercraft wavemaking. Unpublished *N.P.L.* paper.

LAMB, H. 1926 On wave resistance. *Proc. Roy. Soc.* A, **111**, 14–25.

NEWMAN, J. N. & POOLE, F. A. P. 1962 The wave resistance of a moving pressure distribution in a canal. *Schiffstechnik*, **9**, no. 45.

RAYLEIGH, LORD 1883 The form of standing waves on the surface of running water. *Proc. Lond. Math. Soc.* **15**, 69. (Also *Sci. Papers*, **2**, 258.)

WATSON, G. N. 1923 *Theory of Bessel Functions*. Cambridge University Press.